B.Sc. Part - II

Physics.

Maxwell's Field Equation

We now wish to consider the more general ~~equation~~ situation in which the field quantities may depend upon time. Under such conditions there is an independence of the field quantities and it is no longer possible to discuss separately the electric and magnetic fields and we know the general concept of an electromagnetic field. The time dependent electromagnetic field equation are called Maxwell's equation. These equations are mathematical abstractions of experimental results.

There are four Maxwell's equations.

The following four laws of electricity and magnetism constitutes differential form of Maxwell's equations.

(i) Gauss' Law for the electric field of charge yield's

$$\text{div } D = \nabla \cdot D = \rho$$

where D is electric displacement in coulombs/m^2 and ρ is the ~~free~~ charged density/ in coulomb/m^3.

(ii) Gauss Law for magnetic field yields,

$$\text{div } B = \nabla \cdot B = 0$$

where B is the magnetic induction in web /m^2.

(iii) Ampere's law in circuital from for the magnetic field accompanying a current when modified by Maxwell yields.

$$\text{curl } H = \nabla \times H = J + \frac{\partial D}{\partial t}$$

where H is the magnetic field intensity in ampress/m and J is the current density in amp/m^2.

(iv) Faraday's law in circuital form for the induced electromotive force produced by the rate of change of magnetic flux linked with the path yields.

$$\text{curl } E = \nabla \times E = -\frac{\partial B}{\partial t}$$

where E is the electric field intensity in volts/m.

(B) Derivations :—

(1) Let us consider a surface S bounding a volume τ within a dielectric. Originally the volume τ contains no net charge but we allow the dielectric to be polarised say by placing it in an electric field. We also deliberately place some charge on the die-electric body. Thus we have two type of charges :-

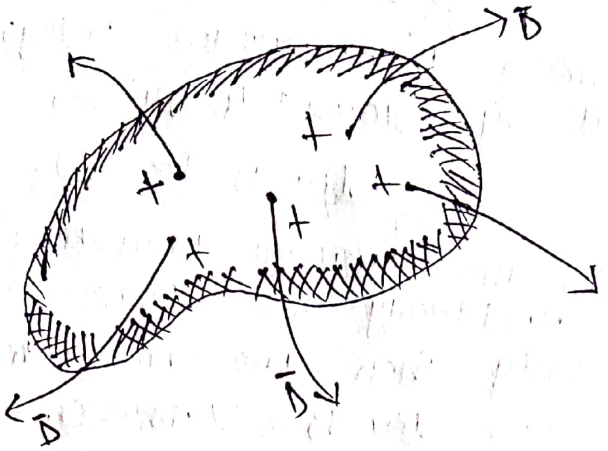
(a) real charge of density ρ (b) bound charge density ρ
Gauss' law then can be written as

$$\oint_S E\, ds = \frac{1}{\epsilon_0} \int (\rho + \rho')\, d\tau$$

$$\epsilon_0 \oint_S E \cdot ds = \int_\tau \rho\, d + \tau \int \rho'\, d\tau. \quad \text{——— (1)}$$

Field Equations and conservation laws
But as the bound charge density ρ' is defined as $\rho' = -\text{div } P$

and $\oint_S E \cdot ds = \int_\tau \text{div } E\, d\tau$



So equation (1) becomes

$$\epsilon_0 \int_\tau \text{div } E \, d\tau = \int_\tau \rho \, d\tau - \int_\tau \text{div } P \, d\tau$$

i.e.

$$\int_\tau \text{div}(\epsilon_0 E + P) \, d\tau = \int_\tau \rho \, d\tau$$

or

$$\int_\tau \text{div } D \, d\tau = \int_\tau \rho \, d\tau \quad (\text{as } D = \epsilon_0 E + P)$$

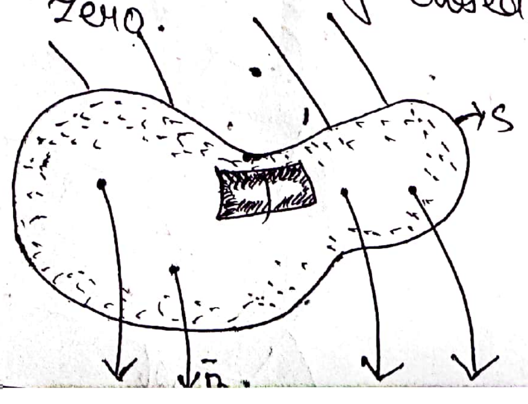
or

$$\int_\tau (\text{div } D - \rho) \, d\tau = 0.$$

Since the equation is true for all volumes, the integrand must vanish. Thus we have

$$\text{div } D = \nabla \cdot D = \rho.$$

(ii) Experiments to-date have shown that magnetic ~~lines~~ monopoles do not exist. Thus in turn implies that the magnetic lines of force are either closed group or go off to infinity. Hence the number of magnetic lines of force entering any arbitrary closed surface is exactly the same leaving it. Therefore the flux of magnetic induction B across any closed surface is always zero.



$$\oint_S B \cdot ds = 0.$$

Transforming this surface integral into volume integral by Gauss' theorem, we get.

$$\int_\tau \operatorname{div} B \, d\tau = 0.$$

But as the surface bounding the volume is quite arbitrary the above equation will be true only when the integrated vanishes i.e.

$$\operatorname{div} B = \nabla \cdot B = 0.$$

(iii) From Ampere's circuital law the work done in carrying unit magnetic pole once round a closed arbitrary path linked with the current I is expressed by

$$\oint_c H \cdot dI = I$$

$$\oint_c H \cdot dI = \int_S J \cdot ds \quad (\text{as } I = \int J \cdot ds$$

where S is the surface bounded by the closed path C. Now changing the line integral into surface integral by Stroke's theorem, we get

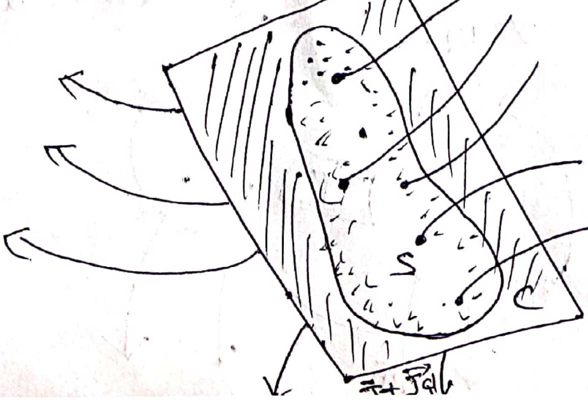
$$\int_S \operatorname{curl} H \cdot ds = \int_S J \cdot ds$$

i.e. $\operatorname{curl} H = J$. ——— (2)

But Maxwell found it to be incomplete for changing electric fields and assumed that a quantity :— $J_d = \frac{\partial D}{\partial t}$ called displacement current must also be included in it so that it may satisfy the continuity equation i.e. J must be replaced in equation (2) by $J + J_d$ so that the law becomes

$$\operatorname{curl} H = J + J_d$$

$$\operatorname{curl} H = J + \frac{\partial D}{\partial t} \quad \text{——— (C)}$$



Continue.