B. Sc. Part – I

Physics

D'Alembert's Principle.

We have already found that if x, y, z be cordinates of a particle m at time t, its motion is found by equating $m\frac{d^2x}{dt^2}$ to the force parallel to the axis of x and simillarly for the motion parallel to the axes of y and z.

If m be a portion of a rigid body its motion is simillarly given, but in this case we must include under the forces parallel to the axes not only the external forces acting on the particle which are due to the actions of the rest of the body on it.

The quantity $m\frac{d^2x}{dt^2}$ is called the effective force acting on the particle parallel to the axes of x

Thus we may say that the x-Component of the effective forces is equivalent to the x-Component of the external forces together with the x-Component of the external forces,

or again that the x Component of the reversed effective forces together with the x Components of the external and internal forces from a system in equilibrium.

So for the components parallel to the axes of y and z.

Hence the reversed effective effective force, the external force and the internal force acting on each particle of the body, the external forces and the internal forces are in equilibrium. Now the internal forces are in equilibrium amongst themselves, for the Newton's third Law there is to every action, an equal and opposite reaction.

Hence the reversed effective forces acting on each particle of the body and the external forces of the system are in equilibrium. This is the D'Alembert's principle.

This principle of the preceding article says that forces whose components are

$$X - m\frac{d^2x}{dt^2} \qquad Y - m\frac{d^2y}{dt^2} \qquad Z - \frac{md^2z}{dt^2}$$

where X, Y, Z are components parallel to axes of the external forces acting on the particle m.

Hence from the ordinary conditions of equilibrium we have

$$\Sigma\left(X - m\frac{d^2x}{dt^2}\right) = 0$$

$$\Sigma\left(Y - m\frac{d^2y}{dt^2}\right) = 0$$

$$\Sigma\left(Z - m\frac{d^2y}{dt^2}\right) = 0$$

$$\Sigma\left[y\left(Z - m\frac{d^2Z}{dt^2}\right) - z\left(Y - m\frac{d^2y}{dt^2}\right)\right] = 0$$

$$\Sigma\left[z\left(X - m\frac{d^2x}{dt^2}\right) - x\left(Z - m\frac{d^2Z}{dt^2}\right)\right] = 0$$

and $$\Sigma\left[x\left(Y - m\frac{d^2y}{dt^2}\right) - y\left(X - m\frac{d^2x}{dt^2}\right)\right] = 0$$

These given

$$\Sigma m\frac{d^2x}{dt^2} = \Sigma X \quad \text{---} \quad (1)$$

$$\Sigma m\frac{d^2y}{dt^2} = \Sigma Y \quad \text{---} \quad (2)$$

$$\Sigma m\frac{d^2z}{dt^2} = \Sigma Z \quad \text{---} \quad (3)$$

$$\Sigma m\left(y\frac{d^2z}{dt^2} - z\frac{d^2y}{dt^2}\right) = \Sigma(yZ - zY) \quad \text{---} \quad (4)$$

$$\Sigma m\left(z\frac{d^2x}{dt^2} - x\frac{d^2z}{dt^2}\right) = \Sigma(zX - xZ) \quad \text{---} \quad (5)$$

and $$\Sigma m\left(x\frac{d^2y}{dt^2} - y\frac{d^2x}{dt^2}\right) = \Sigma(xY - yX) \quad \text{---} \quad (6)$$

These are the equations of motion of any rigid body.

Eqn (1), (2) and (3) states that the sums of the components parallel to the axes of co-ordinates of the effective forces are respectively equal to the sums of the ~~co-ordinat~~ components parallel to the same axes of the external forces.

Eqn (4), (5), (6) states that the sameof moments about the axes of the co-ordinates of the effective forces are respectively equal to the sums of the moments about the same axes of the external impressed forces.

It is aplied for the motion of the centre of inertia, motion relative to the centre of inertia.

Let $(\bar{x}, \bar{y}, \bar{z})$ be the co-ordinates of the centre of inertia, and M the mass of the body.

Then $M\bar{x} = \Sigma mx$ through out the motion, and therefore

$$M \frac{d^2\bar{x}}{dt^2} = \Sigma m \frac{d^2x}{dt^2}.$$

Hence eqn (1) of the last article gives

$$M \frac{d^2\bar{x}}{dt^2} = \Sigma X \quad \text{———} \quad (7)$$

$$M \frac{d^2\bar{y}}{dt^2} = \Sigma Y \quad \text{———} \quad (8)$$

$$M \frac{d^2\bar{z}}{dt^2} = \Sigma Z \quad \text{———} \quad (9)$$

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